HOW MANY STAKES ARE REQUIRED TO MEASURE THE MASS BALANCE OF A GLACIER?

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ABSTRACT. Glacier mass balance is estimated for South Cascade Glacier and Maclure Glacier using a one-dimensional regression of mass balance with altitude as an alternative to the traditional approach of contouring mass balance values. One attractive feature of regression is that it can be applied to sparse data sets where contouring is not possible and can provide an objective error of the resulting estimate. Regression methods yielded mass balance values equivalent to contouring methods. The effect of the number of mass balance measurements on the final value for the glacier showed that sample sizes as small as five stakes provided reasonable estimates, although the error estimates were greater than for larger sample sizes. Different spatial patterns of measurement locations showed no appreciable influence on the final value as long as different surface altitudes were intermittently sampled over the altitude range of the glacier. Two different regression equations were examined, a quadratic, and a piecewise linear spline, and comparison of results showed little sensitivity to the type of equation. These results point to the dominant effect of the gradient of mass balance with altitude of alpine glaciers compared to transverse variations. The number of mass balance measurements required to determine the glacier balance appears to be scale invariant for small glaciers and five to ten stakes are sufficient.

Key words: glaciers, mass balance, statistics

Introduction

When starting a program of surficial mass balance measurements on a glacier, the question inevitably arises regarding the number of observation points required to estimate reliably the yearly mass change. For small alpine glaciers (<20 km^2), typically stakes are placed in a pattern that provides a more or less even distribution of points over the surface (Østrem and Brugman 1991). On larger glaciers (>20 km^2) stakes are typically placed along longitudinal profiles to measure the altitudinal distribution of accumulation and ablation. These distributions are governed by the general belief that a complete and evenly weighted spatial representation is required to assess variations in mass balance. Variations include accumulation differences due to direct snow fall, avalanches, and wind distribution, and include ablation differences due to topographic shading and local sources of dust that reduce surface albedo. The desired distribution is necessarily modified by the realities of surface conditions (steep slopes or highly crevassed) and available resources, either of which may limit the number of observation sites. An example of the effect of available resources is that, very small glaciers tend to have a denser network of observation points (Maclure Glacier, 0.2 km^2, 38 points, 190 points/km^2: Tangborn *et al.* 1977) than bigger glaciers (e.g. Wolverine Glacier, 17 km^2: Meier *et al.* 1971). Furthermore, on large glaciers, because of limited resources, a longitudinal profile of observation sites is established to assess the dominant effect of altitude on accumulation and ablation, at the expense of the lateral variations, which are considered secondary. How do we know when our sample size of observations is sufficient?

A similar question is faced by on-going programs which may want to reduce the number of observation points. After measuring the mass balance for a number of years the experienced glaciologist begins to recognize spatial patterns and eliminates what appear to be redundant observation points. This change reduces the cost of basic data collection and increases the time available for other research studies. However, the apparent redundant information lost may be significant for years of unusual patterns of accumulation and ablation. How many observation points can be eliminated?

These considerations are linked by two basic and interrelated questions. First, how many stakes are required to estimate the yearly mass balance? This is an important question in these times of financial constraints on glacier programs. Second, what is the resulting error in our mass balance estimate? This second question is critical to reliability of mass balance calculations for estimating glacier response to climate and glacier effect on regional water supplies and global sea level. Considering that glaciers are constantly evolving towards an equilibrium condition of zero mass balance, an assessment of errors is fundamental to determining the sign (+/-) of the balance as well as its magnitude. In this light, it is surprising that many mass balance programs do not apply a rigorous error analysis. These two questions are interrelated because, as one can imagine, there is no one right answer to the first question, but rather it depends on the acceptable size of the resultant error.

Perusal of the glaciological literature provides little guidance on the required number of mass balance observations and resulting errors on the mass balance estimate. One of the first reports to address this issue was Campbell (1969). He addressed an erroneous opinion, which is still current, that a fixed density of mass balance observations per unit area is the only statistically reasonable solution for calculating mass balance accurately. This approach, of course, assumes that all glaciers are small alpine glaciers and ignores the vast range of glacier sizes. A sampling density of one observation per square kilometer is too low for alpine glaciers with areas of <1 km^2 (e.g. Andrews Glacier, Front Range, Colorado, USA) and too high for Columbia Glacier of coastal Alaska, USA, with an area of 1100 km^2. Campbell's analysis used basic statistical theory, to determine the number of mass balance observations required to insure, with some degree of probability, that the mean of the observed values lies within a certain deviation of the true mean. This approach, as Campbell points out, is applicable to areas on the glacier where the mass balance values are independent and normally distributed.

Lliboutry (1974) examined the statistics of point mass balance measurements and derived a linear model to account for independent spatial and temporal variations. This approach is well founded in empirical studies that observe relatively constant spatial differences between stakes, but a temporal offset in magnitude of all stakes (Meier 1962; Meier and Tangborn 1965). The model was successfully applied to a set of mass balance measurements in the ablation zone of Glacier de Saint Sorlin covering an area of about 1 km^2 and an altitude range of about 200 m. However, the model was not used to estimate the number and positioning of stakes, nor to estimate the error of the mass balance of the glacier. Lliboutry pointed out that one of the most restrictive assumptions in his theory was that the errors between the model results and observations were normally distributed.

Funk *et al.* (1997) applied Lliboutry's approach to the mass balance of Griesgletscher, Switzerland, to determine whether the number of spatial measurements could be reduced. They found that the differences between the model and measurements better approximated a random distribution if the accumulation and ablation zones were treated separately. To estimate mass balance of the whole glacier a kriging routine was used to interpolate the spatial factor in the equation from specific points to the entire surface of the glacier. The significant correlating factor in the kriging interpolation was altitude, in line with many previous empirical investigations on the changes in glacier mass balance (e.g. Paterson 1994). Once the coefficients in Lliboutry's linear mass balance model are determined, the number of measurement points can be reduced because the spatial changes are assumed independent of time (Lliboutry 1974) and only a few points are required to estimate temporal changes. The data set was reduced to six measurement sites (three in the accumulation zone and three in the ablation zone) with little reduction in accuracy. Funk *et al.* (1997) concluded that a high spatial density of measurements is required to obtain a reliable interpolation function and, once this function is determined, the spatial density can be reduced.

Cogley (1999) examines the effect of correlation among mass balance measurements on a glacier. He found that the measurements closest to each other in altitude were highly correlated and the correlation decreased with increasing elevation differences. This implies that any given measurement is indicative of the region around it and making more measurements in the vicinity will not necessarily add new information. One implication of this work is that dense networks of glacier measurements are probably not necessary and a reduced network, based on the correlation distance among measurements can be implemented.

In this paper we focus on two goals: (1) determine the quality of regression methods for estimating average glacier mass balance compared to the traditional technique of hand-contouring maps of mass balance variations; (2) examine the effects of sample size and the spatial distribution of observations on the resultant magnitude of mass balance and its error. Our approach differs from Funk *et al.* (1997) in that, rather than starting from Lliboutry's (1974) approach, which presupposes a network of measurement sites fixed in space and assumes that the errors in the network are random, we acknowledge and incorporate the dominant effect of altitude on mass balance. We also differ from Cogley (1999), who examines the correlation *between* measurements and their altitude difference and their effective area of representation, by modeling mass balance as a function of altitude. One of our results will show that we are consistent with Cogley's and Funk *et al.'s* conclusion that dense measurement networks are not necessary.

Approach

For most alpine valley glacier environments mass balance is strongly correlated with altitude. The physical process is readily understandable because cooler temperatures with altitude increase snow fall and decrease ablation, in this case melting and sublimation. Although the specific form of the relation between mass balance and altitude is complicated by the meteorological environment and local topography, the correlation and physical rationale of the two are undisputed (Paterson 1994). Exceptions to this generalization include alpine glaciers that are wider than their length or where wind redeposition is of similar magnitude to direct snowfall (Chinn 1985). However, even in these situations mass balance is mildly correlated with altitude. Therefore, we explicitly use this well founded knowledge and assume to the first approximation that mass balance varies with altitude alone,

$$B = \frac{1}{A}\int b(z)\, \mathrm{d}S \qquad (1)$$

where B is the specific balance for the entire glacier expressed as meters of water equivalent (m weq), A is the total glacier area, the integral is over the surface of the glacier, S, and $b(z)$ is the specific balance as a function of altitude where z is a function of (x,y) over the glacier's surface. The function, $b(z)$, is estimated by a least squares fit between point measurements of mass balance and altitude. From the least squares fit, a standard error is calculated which is used to determine the error of the final mass balance value for the whole glacier. The statistical details of this approach are included in the appendix.

To examine the effect of sample size and measurement location on the estimate of mass balance, data points are removed from the sample and a curve is fitted to the remaining data. Theoretically, one might consider randomly choosing points over the surface of the glacier to include in the regression, but the measurements could be clustered in one region (altitude band) of the glacier rather than distributed over the full extent of the glacier's elevation range. From a practical perspective we know that more positive balances occur at the head of the glacier and more negative balances occur near the toe. Therefore, the pragmatic approach is to distribute the measurements over the altitude range of the glacier. This does not imply that the distribution of mass balance measurements is uniform over the glacier surface and leaves much room for different sampling strategies. This approach has an advantage over hand-contoured maps in that an error is objectively determined. This becomes particularly important when fewer and fewer stakes are used, precluding the construction of a contour map whether by hand or by two-dimensional trend analysis.

Application

The analysis was applied to two glaciers. South Cascade Glacier, North Cascade Range of Washington, USA (48°21'N 121°03'W), is a valley glacier flowing from one of the high ragged ridges and into a deeply incised valley typical of much of the topography in this region. At the time of the measurements, the glacier was 2.9 km^2 in area, about 3.5 km long, and ranged in altitude from about 1600 to 2300 m with an equilibrium line altitude at about 1900 m. Winter balance averaged about 3.1 m water equivalent (weq) (Meier *et al.* 1971). Maclure Glacier, located in the Sierra Nevada Mountains of California, USA (37°24'N 119°10'W), is a small cirque glacier (0.2 km^2) located near the crest of the range (Fig. 1). The glacier was about 0.5 km long, ranged in altitude from about 3570 to 3790 m with an equilibrium line altitude of about 3650 m. The winter balance is about 2.3 m weq. (Tangborn *et al.* 1977).

The mass balance data from the two glaciers are yearly net mass balance using the combined stratigraphic and fixed-dates systems (Mayo *et al.* 1972) derived from accumulation and ablation measurements made at stakes drilled into the glacier surface. The estimated value of total glacier mass balance was calculated from a hand-contoured map of interpolated values across the glacier surface. The details of the mass balance methodology and results are summarized in Meier *et al.* (1971) and Tangborn *et al.* (1977). These two glaciers were chosen because for several years they each had a large number of point mass balance measurements, peaking at 38 on each glacier, and the area and

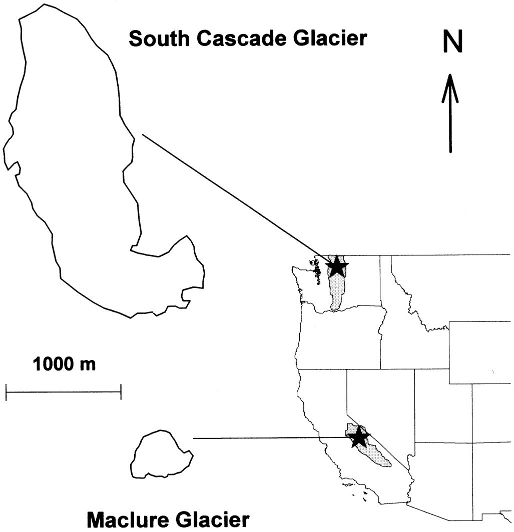

Fig. 1. Location map and relative sizes for South Cascade and Maclure glaciers. The scale bar applies to both glaciers. The shaded areas on the map highlight the mountain range in which each glacier is located. South Cascade Glacier is located in the North Cascade Range and Maclure is located in the Sierra Nevada Range.

point density of the glaciers differ by an order of magnitude, 2.9 km^2 (13 stakes km^{-2}) for South Cascade and 0.2 km^2 (190 stakes km^{-2}) for Maclure. The relatively high density of mass balance observations allows us to observe the effect of reducing the density and altering the spatial pattern on the resulting mass balance value for the entire glacier. We compare our estimates of glacier mass balance to the original mapped values calculated by Meier *et al.* (1971) and Tangborn *et al.* (1977). The values of mass balance at each stake are expressed as meters of water equivalent (m weq).

To describe the variability of mass balance with altitude we used two models, a quadratic model

$$b(z) \;=\; \beta_0 + \beta_1 z + \beta_2 z^2 + \varepsilon \qquad (2)$$

and a piecewise linear spline with weighted errors

$$b(z) \;=\; \begin{bmatrix} \beta_0 + \beta_1(z - z_0) + w_1\varepsilon,\; z \le z_o \\ \beta_0 + \beta_2(z - z_0) + w_2\varepsilon,\; z \ge z_o \end{bmatrix} \qquad (3)$$

In each model, $ß_i$ is a constant estimated by the least squares fit, z is the altitude of the glacier surface and is a function of (x,y), z_o is a datum altitude used to scale the magnitude of the altitudes about 0 for a more robust statistical evaluation of the errors, and ε is the random error about the fit.

The quadratic equation was used because this simple polynomial appears to describe many mass balance–altitude curves (see Meier *et al.* 1971 for examples) and has been used previously to model mass balance with altitude (Konovalov 1987; Tangborn *et al.* 1990). The piecewise-linear spline was used to explore the effect of a different model on estimated mass balance. In addition, weights were included in the spline to explore the effect of non-constant error variances on estimated mass balance. Measurement error may differ depending on whether the accumulation zone or ablation zone is being sampled and we wanted to examine possible effects from such differences.

Data sets

For South Cascade Glacier we used the 1965 (38 points) and 1966 (24 points) data from Meier *et al.* (1971) and the 1968 (25 data points) data from Tangborn *et al.* (1977). For Maclure Glacier we used the 1967 data from (Tangborn *et al.* 1977), which had 38 observation points.

Results

South Cascade Glacier

The stake distributions for the years 1965, 1966, and 1968 are shown in Fig. 2. The stakes cover the glacier in a relatively even distribution with a gap near the middle where a large icefall is present. Using the quadratic equation and all the stakes for each year, the comparison between the estimated and mapped value is shown in Table 1. A plot of the quadratic curve and measured mass balance points is presented for 1965 in Fig. 3.

The errors for the mapped and estimated values are different. For the mapped value, the error is estimated from the field measurements for each stake and assumed the same for all stakes. No estimate of

Table 1. A comparison between estimated and mapped value for mass balance of South Cascade Glacier. The root mean squared error is also shown (±).

Year	Number of stakes	Mapped value (m weq)	Estimated (m weq)	Difference (m weq)
1965	38	-0.17 ± 0.12	-0.13 ± 0.12	+0.04
1966	24	-1.03 ± 0.12	-1.08 ± 0.22	-0.05
1968	25	0.01 ± 0.12	0.00 ± 0.33	-0.01

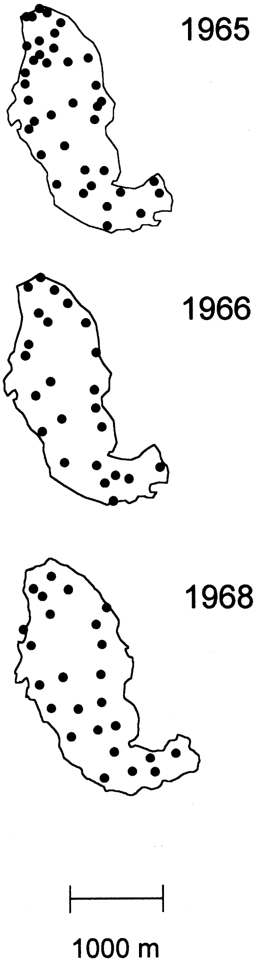

Fig. 2. Stake locations on South Cascade Glacier for three different years.

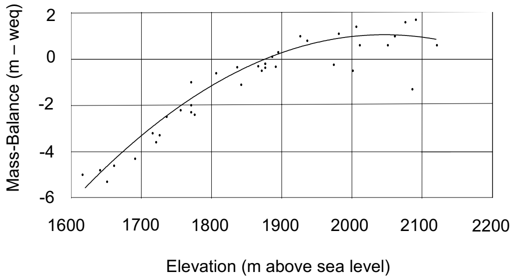

Fig. 3. An example of the quadratic equation fit to all the stakes measured in 1965 on South Cascade Glacier. The abbreviation, weq, is water equivalent of snow or ice.

the error from contouring the mass balance data was made. For the estimated values, the error is based on the 95% confidence interval, as described in the Appendix. The resulting values of mapped and estimated mass balance are close to each other and the difference between the two is at least a factor of two smaller than the smallest error.

Effect of sample size. To determine the effect of sample size on the estimated value of mass balance for the glacier, using Equation 1 and the quadratic Equation 2, stakes were selected from the total population in increments of five up to the total number of observations available. Sets of less than five were regarded as too small for the regression technique. Within each sample size category, different groups of stakes were selected according to the practical strategy of covering as much of the altitude range of the glacier as possible. The results are presented in Fig. 4 where the estimates and 95% confidence intervals are plotted against the number of stakes.

Two features are immediately apparent. First, the confidence interval decreases with increasing sample size, and second, the predicted mass balance values using the smaller data sets are generally close to the value using the whole data set. Smaller data sets produce more variance in the estimates and exhibit larger confidence intervals. Given a fixed upper limit to the number of stakes a smaller sample can be rearranged in many different ways compared to a larger sample. From another perspective, different sets of large samples contain many of the same stakes and the relatively few stakes that are changed have comparatively little influence on the result. A reason for the comparable mass balance values from different sample sizes results from the stake selection, which was not random, but instead was designed to obtain a reasonable distribution with altitude. Within this distribution attempts were made to select stakes more-or-less randomly across the glacier width. We found no consistent influence of the lateral variation in mass balance on the value of total mass balance of the glacier. This result indicates that the mass balance gradient with altitude exerts a much stronger influence on the total glacier mass balance than do lateral variations.

Table 2 summarizes the data presented in Fig. 4 and shows that the mean difference between the mass balance estimate using all stakes and the estimates based on smaller sample sizes shows no consistent relation. The root mean squared error of the estimates generally increases with smaller sample sizes.

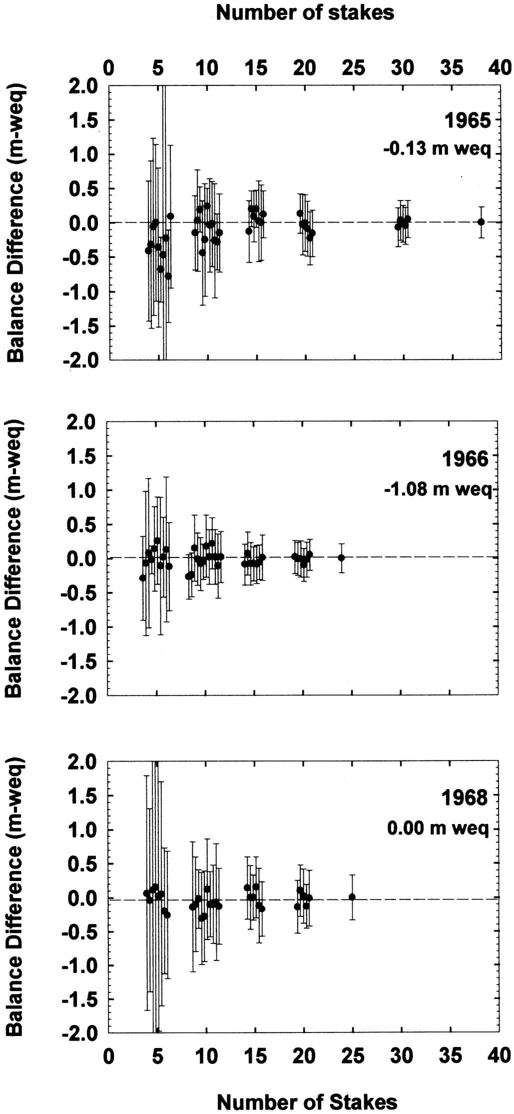

Fig. 4. The difference between the mass balance calculated from samples of different sizes and that calculated from all the stakes. The error bars represent the 95% confidence intervals. The slight spreading of points was introduced to avoid overlap. The mass balance value in the upper right hand corner is the mapped mass balance estimate by the original authors of the mass balance study. The abbreviation, weq, is water equivalent of snow or ice.

Effect of sampling location. Tests were run with different stake arrangements to determine if some arrangements estimated the mass balance better than others. In test runs of sets with five stakes we tried a pattern of two stakes in the ablation zone, one near the equilibrium line, and two stakes in the accumulation zone to try and capture a measure of the natural variance in mass balance values, but these tests resulted in large differences in mass balance values. The differences between the curves of mass balance with altitude (not shown) of the more evenly distributed stakes with elevation and the emphasis on the upper and lower ablation zones was largest in the central portion of the glacier. The distribution of glacier area with altitude for South Cascade Glacier is greatest around the central portion of the glacier and decreases towards the upper and lower ends. Thus, this distribution of area enhanced the errors incurred in not properly delineating the trend of mass balance with altitude.

Table 2. Mean difference from mass balance estimate using all stakes and root mean squared error of estimates computed. The root mean squared error is also shown (±).

Number of stakes	1965 Mean difference (m)	1966 Mean difference (m)	1968 Mean difference (m)
5	-0.32 ± 0.41	0.00 ± 0.15	-0.01 ± 0.17
10	-0.10 ± 0.22	-0.01 ± 0.23	-0.11 ± 0.23
15	0.07 ± 0.13	-0.05 ± 0.09	0.00 ± 0.13
20	-0.07 ± 0.13	-0.01 ± 0.05	-0.03 ± 0.10
30	-0.01 ± 0.05		

The runs using sets of five stakes we present here are those that are arranged in a longitudinal fashion up the glacier. We selected stakes without regard to their lateral position across the glacier. We found that the lateral position did not influence the estimated value of mass balance in any consistent way. For sets with 10 and 15 stakes, three different patterns were tried, including a uniform scatter of stakes, two parallel lines of stakes, and a single line of stakes in the ablation zone with a uniform scatter in the accumulation zone. The latter pattern was inspired by the observation that the mass balance seems to increase in a near-linear manner in the ablation zone and in the accumulation zone a less strong relation between altitude and mass balance exists (Fig. 3). This pattern is common to the three years of data. None of these different sampling strategies consistently improved the estimated mass balance. The number of cases analyzed for each pattern and for each set size varied from two to seven. Although this small test is not statistically robust and many more cases from other years at South Cascade or from other glaciers need to be tested, it suggests that there is no significant difference in mass balance estimates from changing lateral placement of the stakes.

The data were also examined to determine whether some stake locations were particularly poor or good in estimating total glacier mass balance. The suitability was determined by the deviation of the stake measurement from the quadratic curve fit to all points. Those stakes that deviated by more than 0.5 m weq from the curve were considered 'poor' and those that deviated less than 0.25 m weq from the curve were considered 'good'. Although we found 'good' and 'poor' stakes they were not representative of an area on the glacier. For example, we found 'good' and 'bad' stakes along the glacier margin, in the center, in the accumulation zone and in the ablation zone. Moreover, we did not find any consistency between years.

Effect of equations. In addition to the quadratic equation (Equation 2), a piecewise linear spline (Equation 3) was chosen because the data could be modelled by two straight lines. Also, the effect of different weights on the data for the accumulation zone and ablation zone could be investigated. Although the errors given in Meier *et al.* (1971) are constant for all stakes, we applied a weight of 1 for the ablation zone and $[2]^{1/2}$ for the accumulation zone, giving the latter twice the variance of the former. An example of the piecewise linear spline is shown in Fig. 5. Table 3 shows that the piecewise linear estimates, both weighted and unweighted, yield results nearly equivalent to those from the quadratic equation (the unweighted case is obtained by setting $w_1 = w_2 = 1$). Weighted regression was also tried for the quadratic model (not shown) and yielded results quite similar to the unweighted model. Like the results with the quadratic equation alone, the standard error decreases with increasing sample size. For the spline equation, the investigator needs to define the location of the break point between the linear segments of the spline. For the case that included all points, the location of the break point producing the best fit coincided with the equilibrium line altitude. However, the result is not particularly sensitive to the location of the break point.

Maclure Glacier

The spline and quadratic equations were applied to Maclure Glacier to test our approach on a different glacier. Maclure Glacier is an interesting case because it is quite small (0.2 km^2) and the 1967 mass balance study (Tangborn *et al.* 1977) installed 38 stakes yielding a stake density of 190 stakes/km^2.

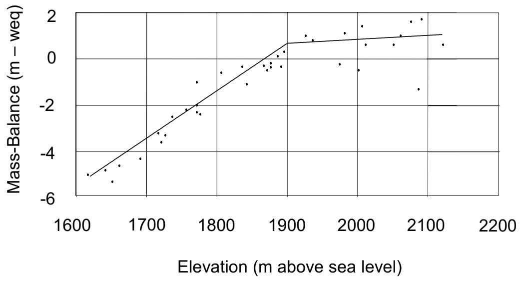

Fig. 5. Example of the piecewise linear spline fit to all the stakes measured in 1965 on South Cascade Glacier. The abbreviation, weq, is water equivalent of snow or ice.

Also, because this glacier is a small cirque glacier, the variance of mass balance in the transverse direction is high and the altitudinal gradient is not as strong as South Cascade Glacier. Fig. 6 shows the quadratic fit to all the stakes measured in 1967. Immediately apparent is the scatter of the points. Results of the quadratic and spline fits are shown in Table 4. It is clear that the quadratic and spline estimates and errors are very similar to each other. Doubling the variance for the weighted spline has little effect on the result. In comparison to South Cascade Glacier, the errors are higher as one might expect from the variance of the data in Fig. 6. The mapped value of the mass balance, based on the hand-contoured map, is 1.10 m weq, which is close to the values given in Table 4.

To apply the spline and quadratic equations, we assumed that the deviation from the curve (residuals), ε, was uncorrelated white noise. Using likelihood ratio tests as described by Vecchia (1988), we tested for spatial correlation between the residuals. For the two data sets, the 1965 data from South Cascade, and the 1967 data from Maclure Glacier, no spatial correlation was found in ε.

Table 3. Mass balance estimates for different sample sizes of the 1965 data for South Cascade Glacier using different equations. The root mean squared error is also shown (±).

Number of stakes	Quadratic (m weq)	Spline no weights (m weq)	Spline with weights (m weq)
5	-0.52 ± 0.48	-0.52 ± 0.43	-0.51 ± 0.36
10	-0.36 ± 0.26	-0.39 ± 0.25	-0.40 ± 0.23
20	-0.29 ± 0.18	-0.28 ± 0.17	-0.29 ± 0.16
38	-0.13 ± 0.12	-0.12 ± 0.11	-0.12 ± 0.11

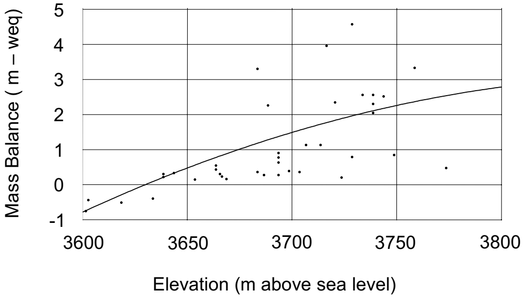

Fig. 6. Example of the quadratic fit to all the stakes measured on Maclure Glacier in 1967. The abbreviation, weq, is water equivalent of snow or ice.

Discussion and conclusions

Application of regression equations, based on altitude, to glacier mass balance data for the purpose of estimating the total mass balance is a robust approach. The quadratic and piecewise linear spline models produce similar results that are equivalent to mass balance values estimated by the traditional mapping approach. The effect of stake number, down to five stakes, shows surprisingly little variation about the mapped value of mass balance. Such little variation probably results from the strong dependence of mass balance on altitude and the small transverse variation. However, the error increases with decreasing sample size. We conclude that the regression approach can be used with all sample sizes down to five stakes with decreasing reliability.

From our analysis of three different years of data from South Cascade Glacier and one year of data from Maclure Glacier, we conclude that five to ten stakes per glacier are suitable for small alpine glaciers (*c.* <10 km^2). This result supports that of Funk *et al.* (1997) who found six stakes were suitable for Griesgletscher (6.2 km^2). Increasing the stake number will decrease the root mean squared error

Table 4. Mass balance estimates for different sample sizes using different equations and the 1967 data for Maclure Glacier. The root mean squared error is also shown (±).

Number of stakes	Quadratic (m weq)	Spline no weights (m weq)	Spline with weights (m weq)
5	1.52 ± 0.48	1.50 ± 0.47	1.61 ± 0.49
10	1.52 ± 0.41	1.52 ± 0.42	1.51 ± 0.43
20	1.23 ± 0.29	1.22 ± 0.30	1.23 ± 0.30
38	1.28 ± 0.24	1.28 ± 0.24	1.30 ± 0.23

of the result but the actual value of the result will probably not change appreciably. Thus, maximizing stake density is not necessarily an important guide when assessing the appropriate number of stakes on a glacier. This conclusion supports Cogley (1999) who found a high correlation for stakes within a narrow elevation band, implying that little new information about the variance of mass balance is added by more stakes in close proximity to each other. Taken together, these results strongly indicate that only a small number of stakes are required to estimate the mass balance of a glacier. It is prudent, however, to compare the estimated mass change over a number of years with that derived from volume change estimates based on geodetic methods (Krimmel 1989). Such a check is particularly important for glaciers with small stake networks to reduce the influence of year-to-year variance in mass balance error on the total mass change of the glacier.

Three different patterns of stake arrangements showed little consistent effect on the estimated mass balance. These patterns were intended to capture the variance of the stake measurements about the regression line while maintaining a distribution over the altitude range of the glacier. The lack of influence of the stake pattern underscores the dominating effect of the altitude gradient of mass balance. We also did not find any consistent patterns of ‘good’ and ‘poor’ stake locations. We speculate that avalanching and wind redistribution of snow are the primary factors causing some stakes to be worse indicators of glacier mass balance than others. Because the stakes were not placed back in exactly the same spot every year, ‘good’ and ‘poor’ locations can not be explicitly identified. The apparent lack of persistence of ‘good’ or ‘bad’ areas suggests that exact repositioning of measurement sites is not important and the choice of appropriate stake locations, particularly important for small sample sizes, is not significant. One is left with the ‘luck of the draw’.

Two different regression equations were used to predict mass balance as a function of altitude. Results showed that the estimated mass balance of the glacier did not significantly change. The common characteristic for all regressions was the ability to include the change in mass balance gradient between the ablation and accumulation zones. For the piecewise linear regressions, the best fits had a break in slope that coincided with the position of the equilibrium line altitude.

The analysis of Maclure Glacier, a glacier that is an order of magnitude smaller than South Cascade Glacier, showed striking similarities with regards to effects of stake number and pattern on the final mass balance values. This is a particularly interesting finding because one would naturally assume that larger glaciers require more mass balance observations than smaller glaciers. Our results indicated that the number of required measurements may be scale invariant. From a stake density perspective, we suggest that larger glaciers need a lower stake density and smaller glaciers require a higher one. We speculate that the mass balance values of cirque glaciers are highly variable due to edge effects which are related primarily to avalanching, wind redistribution, and, to a lesser degree, to ablation enhanced by re-radiation of energy and advection of heat from adjacent solar heated rock. To quantify the variation adequately requires a higher stake density than for larger glaciers where the importance of edge effects is reduced.

There must be some upper limit to this scale invariance or else we could monitor large ice sheets with only five to ten stakes. From a mathematical perspective, we have limited our study to situations where the changing mass balance with altitude can be approximated by a relation that contains no more than one curve (curvature of only one sign). For mass balance–altitude relations that are more complex – containing multiple curves (curvatures of different signs) – our approach requires different equations than presented here. From a climatic perspective, our approach applies to glacial situations where the relation of mass balance with altitude changes only once at the altitude where the boundary between the ice and snow is crossed. For large glaciers exposed to different climates, which may include radically different solar insolation values or accumulation rates (due to differences in precipitation and/or avalanching), the mass balance with altitude relation changes multiple times. In this situation we need to consider other spatial effects on the glacier climate different from our simple focus on altitude alone. Similarly, the mass balances of different parts of an ice cap or ice sheet are affected by their position relative to storm tracks in addition to the effect of altitude, localized wind regimes, and so on, which renders our simple one-dimensional approach inadequate. Thus, we anticipate that the apparent scale invariance breaks down for situations that are not well described by the quadratic or two-segment piecewise linear spline approach.

For estimating glacier mass balance, the regression approach provides an excellent alternative to contouring methods. The contouring approach by an experienced investigator familiar with the local nuances of spatial variations in mass balance may provide a better estimate of the mass balance compared to regression methods. However, contouring is limited to large data sets and an error estimate is ambiguous. The regression approach has the advantage that it can be applied to small as well as large data sets and it provides an objective estimate of the error. This last point is particularly important because an error estimate of mass balance values is often lacking in the literature and it is particularly crucial to glacier studies where values trend towards zero.

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Appendix

Methods are presented for estimating the mass balance of a glacier based on point measurements of mass balance from stakes scattered over the surface of the glacier. The first section establishes some notation and the second section describes computational methods for obtaining the estimates and associated confidence intervals.

Notation

A	area of the glacier (in m^2)
s,u	specified locations on the glacier surface
$z = z(s)$	altitude of the glacier surface at location s (in m a.s.l.)
$b(z)$	mass balance measurement at location z (in m weq)
z_i	altitude of the ith mass balance measurement (i = 1,2,....,n)
$< f(z) > \; = \; A^{-1}\int f(z)\mathrm{d}S$	average of function $f(z)$ over the glacier surface
n	number of point measurements of mass balance
$B = \langle b(z) \rangle$	mass balance for entire glacier (in m weq)
$H(z) = 1$, if $z \le z_o$	indicator function for use in regression analysis, where z_o is a
$= 0$, if $z > z_o$	specified altitude.
$\varepsilon = \varepsilon(s)$	Gaussian, white noise spatial process with mean 0, variance σ^2 and correlation of [$\varepsilon(s)$, ε(u)] = 0 for $s \neq u$.
β_o, β_1,β_2	linear regression parameters
w_1, w_2	weights used in regression analysis

Statistical Methods

Based on examination of the relation between mass balance measurements and altitude for the South Cascade and Maclure glaciers, two models were postulated for describing the spatial variability of $b(z)$ over the glacier surface. The first model is the quadratic model

$$b(x,y) \;=\; \beta_0 + \beta_1 z + \beta_2 \; z^2 + \varepsilon \tag{A1}$$

and the second model is the piecewise linear spline with weight errors

$$b(z) \;=\; \beta_0 \;+\; \beta_1 \, H(z)\,(z \;-\; z_0) \;+\; \beta_2[1-\; H(z)](z \;-\; z_0) \;+\; \{w_1 H(z) \;+\; w_2\;[1\text{-}H(z)]\}\varepsilon \tag{A2}$$

where Equation A-2 is equivalent to Equation 3, but written in a form that is easier for developing the methods of this Appendix.

The goal is to estimate the total mass balance, B, based on n observations $b(z_1)$, ...,$b(z_n)$. The total mass balance can be written using Equation A-1 as

$$B_1 \;=\; \beta_0 \;+\; \beta_1 \;<z>\; + \;\beta_2 <z^2> + \;<\varepsilon> \tag{A3}$$

where $<z>$ is the average elevation and $<z^2>$ is the average squared altitude. The average elevation needs to be known. In this paper, gridded altitude data were used to compute approximate values for $<z>$ and $<z^2>$. Similarly, the total mass balance using Equation A2 is given by

$$B_2 \;=\; \beta_0 \;+\; \beta_1 < H(z)\,(z \;-\; z_0) > \;+\; \beta_2 <[1-\; H(z)](z \;-\; z_0) > \;+\; < w_1 H(z) \;+\; w_2\;[1-H(z)]\varepsilon > \tag{A4}$$

It can be shown using standard methods from spatial statistics (e.g. Cressie 1991, Ch. 3) and the assumed statistical properties for the spatial process, $\varepsilon(s)$, that the best linear unbiased estimate of mass balance corresponding to Equation A-3 is given by

$$\hat{B}_1 \;=\; \hat{\beta}_0 \;+\; \hat{\beta}_1 < z > \;+\; \hat{\beta}_2 < z^2 > \tag{A5}$$

where, $\hat{\beta}_o$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are the ordinary least-squares regression estimates of the parameters using the quadratic model Equation A1. The standard error of the estimate is given by

$$SE(\hat{B}_1) \;=\; \{\hat{\sigma}^2/A \;+\; x^T \; V x\}^{1/2} \tag{A6}$$

where $\hat{\sigma}$ is the standard error of the regression, x is a column vector with elements (1, $<z>$, $<x^2>$), superscript T denotes transpose, and V is the variance-covariance matrix of the estimated regression coefficients. Similarly, the best linear unbiased estimate of mass balance for the piecewise linear model is given by

$$\hat{B}_2 \;=\; \hat{\beta}_0 \;+\; \hat{\beta}_1 < H(z)(z \;-\; z_0) > \;+\; \hat{\beta}_2 <[1-\; H(z)](z \;-\; z_0) > \tag{A7}$$

where $\hat{\beta}_o$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are the weighted least-squares regression estimates using the model Equation A2. The standard error of the estimate is

$$SE(\hat{B}_2) \;=\; \{\hat{\sigma}^2(w_1^2 A_1 \;+\; w_2^2 A_2)/A \;+\; x^T \; V x\}^{1/2} \tag{A8}$$

where A_1 is the area of the glacier below altitude z_o, A_2 is the area of the glacier above elevation z_o, and x is a column vector with elements (1, $<H(z)(z–z_o)>$, $<[1–H(z)](z–z_o)>$).

The results above can be used to compute confidence intervals (prediction intervals) for the mass balance. For example, a 95% confidence interval is computed as follows

$$[\,\hat{B}_i \;-\; t(0.025, n-3)\; SE(\hat{B}_i),\;\; \hat{B}_i \;+\; t(0.025, n-3) SE(\hat{B}_i)\;] \tag{A9}$$

where $t(0.025,n–3)$ is the 0.025 quantile of the t-distribution with n-3 degrees of freedom.